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ANALYSIS AND FORMS OF STRUCTURES CRITICAL STATES

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ABSTRACT: The paper presents certain summary, and important extension of results obtained after long investigations led from years by the present author and his co-workers over instability and critical states of different structures. The essential works were done analytically and by some original own numerical methods with application of Finite Differences, including 3D-Time Space Method (**3D-TSM** – the four dimensional space). So, they provide observations from range of stability and dynamical stability of structures. There, key-part play formulated by Obrębski uniform criterion (used for determination of geometrical changeability and stability of structures). It can be applied as well for classical Euler's, Wagner's, Vlasov's tasks and too many others, for single straight bars as also for complicated structures composed from many different types elements (in it mixed and e.g. plates, too). Many previous examples were presented in the past on LSCE and on world-wide conferences. Application of **3D-TSM** brings a lot interesting possibilities. Calculated non-conventional examples permits to present some new observations, definitions and graphical interpretation of critical states of structures. The method has as usually some limitations, too.

Keywords: Analysis, critical states, forms, uniform criterion, combined loadings, travelling masses, multicriterial instability.

1. INTRODUCTION

The present author, from years, step by step has leading own non-conventional investigations over different questions from domain of *structures critical states* (**SCS**). These works were done analytically in range of straight *thin-walled bars* (**TWBs**) Refs 23, 27, numerically for complicated space bar structures Refs 20, 8, plates, even shells and in hybrid way. There were taken into consideration different manners of boundary conditions (supports) and various types of external loadings including whole sets of combined loadings in it also moving masses - groups of vehicles travelling over bridges, road belts or airport landing strips. Presented conclusions from mentioned examples are collected on basis of large own papers presented during almost all LSCE conferences and on IASS, SEMC (Cape Town) and on few other international conferences. Especially, formulation of *Uniform Criterion* (**UC**) opens many new horizons in domain of structures critical states. There, mathematic key-operation is comparison to zero main determinant Δ of *stiffness matrix* (**SM**) of the various types' tasks (as well by analytical approaches as by numerically advanced ones). In the beginning of present paper is given short review of approaches to the phenomenon, and next are related shortly own investigations and examples from discussed domain. At last, the author presents late, new wide, comparative results concerning of moving loads on bridges. In this way the theses formulated in the beginning for **UC**, are completed by important, new serious and integral proof. It complete in important way, wide investigations led through year's Refs 44, 72-74.

2. METHODS FOR DETERMINATION OF CRITICAL STATES

In literature we can observe during centuries, many diverse approaches to seemingly other kind - different tasks for **SCS**. But, after detailed, closer observation, we can come to conclusion, that all these methods have common criterion of evaluation structures critical states.

2.1. Analytical approaches to instability of structures

In all even historical approaches, the tasks were coming down to comparison to zero certain, one or sets, of differential equations. They were derived very often in extremely some other ways, taking into consideration less or more advanced assumptions and theories, with various structure behaviour, its mechanical properties, boundary conditions, kind of external loadings and interaction with surrounding medium. In very short review, on the ground of a few more representative examples accessible in very wide literature, we can point below following typical solutions of tasks for **SCS**.

Critical compressing axial forces for bended bars with a few different boundary conditions were determined by Leonard Euler (1707-83), Refs 9, 17. There the bar has constant full *cross-section* (**CS**). The one derived differential equilibrium equation for statics only, is of fourth order. Finally, there was compared to zero certain expression taking into consideration bar boundary conditions, too.

In similar way as above were proceeded tasks in "older" literature for determination frequencies of own vibrations, Refs 10, 18.

In academic manual addressed for students, written by P. Jastrzębski (1925-97), J. Mutermilch (1903-90) and W. Orłowski, Ref. 9, are presented very easy solutions for Euler's tasks for compressed bars.

Then, in the encyclopaedic edition by Sylwester Kaliski (1925-78), Ref. 10, are presented some rather complicated, advanced solutions. In the "part two" by W. Bogusz are presented tasks for stability of nonlinear structural systems. There, we can find notions – definitions of: local, complete, absolute and technical stabilities. There, were discussed particular types of equations, for tasks very far from civil engineering applications. Are there presented also solutions for loads moving along straight beam.

Special attention can be turned on stability of frames and trusses presented by Witold Nowacki (1911-86), Ref. 18. There, e.g. for continuous beams on a few supports, the task is also coming down to comparison to zero (*main*) determinant of the sets of homogenous equations. The same condition is obtained for frames and trusses. There, mentioned equations are obtained in different, mathematically advanced ways. From technical point of view presented examples are rather too simple.

Some steps ahead (from technical point of view) are done for **TWBs**, in the book by V.Z.Vlasov's (1905-58), Ref. 82, see also Refs 9, 17, 23. There, are used the sets of three differential equations – two for bending and third for torsion. Applying three displacement functions fulfilling boundary conditions was obtained determinant and its value (in form function of longitudinal loading) finally compared to zero. In effect were calculated critical loadings of bending-torsion character.

In general case, Vlasov has obtained the curve called as "*izostaba*", collecting localizations of longitudinal force, were theoretically instability of the bar is impossible. But Vlasov has commented it contrary – just incorrectly.

Especially very clear explanation of above approach, were given by J. Mutermilch (1903-90) and A. Kociołek, Ref. 17.

Further, Obrębski in manual Ref. 23 has given example of hybrid determination critical states for bar loaded by longitudinal force P and vibrating with frequency ω. Next, in the Refs 63, 67, 68 are given examples various, combined loadings – longitudinal and transversal giving together critical states. In consequence, were defined and shown *ultimate critical curves* and *ultimate critical surfaces*. At last, in Ref. 73 was Investigated influence on critical state of bridge: its length and loadings travelling with certain velocity.

2.2. Numerical methods determination of critical states

All analytical methods of solution mechanical tasks are limited rather to very simple structures, with strongly narrow practical assumptions. There, often scientific discussion is very far from reality. Truth revolution in domain of applied, technical solutions, follows after introducing computer technology. Even here, we can observe numerous varieties of possible calculation technologies. Some of them, applied by present author, are commented below.

Solution of the set linear algebraic equations. One of the mentioned methods is sequential Gauss eliminations, very useful and popular. But in domain of analysis of **SCS**, very important is Cramer's method (see e.g. Ref. 23). There set of linear algebraic equations in the form: $a_{ij}x_j=b_i$ where $i,j=1,2,3,..., n$, has only one solution $x_j=D_j/D$, and $D=\Delta$ is the main determinant of the coefficients (or functions) matrix. Determinant D_j is obtained replacing in D the j-th column by column of free terms b_i. So, the case, when

$$D=0 \text{ means } x_j \rightarrow \text{infinity}. \quad (1)$$

Application of the Finite Element Method (FEM)

The method has very wide literature. It can be applied in some different ways, and here will be not discussed. It is not exact.. (see Ref.76).

Application of the Finite Differences Method (FDM)

There, exists some ways on obtaining the ***F****inite* ***D****ifferences* ***O****perators* (**FDO**) describing e.g. internal forces for bars or matrix equilibrium equation of the whole structure:

$$Kx=Q, \quad (2)$$

Above equation is in reality the set of linear algebraic equations. The method can be applied in some different ways, too. In one of them, the traditional differential equations are formally transformed into **FDO** (see e.g. Ref. 27, 29, 43).

Application of the Difference-Matrix Equations Method (DMEM)

Also this method can be applied in some different ways. The first time it was used for calculation of plane hexagonal grids, Ref. 19, and next, extended on complicated space bar structures, Refs 20, 21, 23, 5. In this description we obtain the global equilibrium equation of whole structure Eqn 2 as set of linear algebraic equations. There, for bar structures we have for each node identical number unknown displacements, as in FEM.

Application of commercial programs. The present author has using in most cases own, elaborated (written) by him selves programs, or sometimes ROBOT Millennium or MS Excel.

2.3. Hybrid methods for determination of critical states

An example of hybrid method application is the task on dynamical-stability heavy steel beam – loaded by longitudinal force and freely vibrating with frequency ω, (Example 11.1, Ref. 23). In hybrid solutions, the derived equations (differential or FDO) are elaborated (transformed, e.g. reduced number of unknown as in plates) analytically and next, last step is executed finally by computer.

2.4. 3D-Time space method

In this approach it is assumed, that solutions concern of the space extended to the four dimensions – three for 3D space and the fourth for time. The idea was developed by several authors and in Poland especially by Z.Kączkowski (1921-) Refs 11-13 and his co-workers e.g. Refs 7, 14, 75, 80, 81. Wider literature for previous such investigations can be found in pointed above references. Indicated here publications are oriented by Kączkowski on application of Finite Element Method (FEM) extended on the fourth dimension – the time. Such approach was named by its originator (Ref. 13) as MECZ (shortening - first letter in Polish). The solved in these way examples were rather technically simple.

Certain step ahead was done in the doctor theses of R.Szmit, by dynamical analysis of tall buildings, Ref. 79. There, were taken into consideration four differential equations, transformed to *Finite Differences Operators* (**FDO**). All was described as in *3D-Time Space Method* (**3D-TSM**). Simultaneously, the method was applied by its originator to masses (forces) travelling on bars (bridges), plates (airport **landing belts or roads), Refs 38, 46.**

As the next important step it was application such description to **stability and dynamical stability** of the different structures: columns and bridges behaviour under mowing loads, Refs 27, 35, 41, 42, 71-73. In the last case, the 3D-TSM was used for testing behaviour of some bridges under travelling masses with wide spectrum of velocities. First time it was presented in Ref. 44 (2004).

3. UNIFORM CRITERION

On importance investigation of main determinant Δ of whole structure was turning the book, Ref. 2. There it was written, that: "by numerical calculations of structures, value (*main determinant*) of coefficients matrix of set of equilibrium equations written (*composed*) for all nodes (*of frame or truss*), must be different from zero". "Contrary, it means geometrical changeability (**GC**) of space bar structure".

Now, we can express conviction, that it is exact method of solution of instability problems, the most ingenious and efficient. So, value of main determinant of coefficients forming stiffness matrix K of structure equal zero $\Delta=\det[K]=0$ prove its geometrical cheangeability (compare Eqn 1) or structure critical state.

In the book Ref. 2, it was commented, than (that time 1970) the approach is time-consuming (structure with 238 nodes and 792 bars was analysed by program written in ALGOL 60 in time of 90 minutes

Just the method was applied in author's computer programs WDKM and KMT, Refs 20, 8. There, possibility of **GC** for space bar structures was checked (tested) on two stages. On the first computer program has checking in local sense postulates formulated in Ref. 20 and quoted in Ref. 8, too. It points probable reason. Next, on second stage, after completion of **SM** - **K**, while – by the way of determination unknown displacements when solving set of linear algebraic equations. So, applying the method of sequential Gauss eliminations was calculated value $\Delta=\det[K]$ for **SM** of whole structure according to formula, Ref. 18:

$$\det[K]= K_{11}\, K^{1}_{22}\, K^{2}_{33} \,.......K^{l-1}_{ll}.......K^{n-1}_{nn}\,, \quad (3)$$

It is a product of terms standing on main diagonal modified stiffness matrix after "n" eliminations, in row with number "n" (with some restrictions), see Ref. 20.

After own experiences and review accessible solutions in wide literature provided by centuries, Obrębski has formulated the *Uniform Criterion* (**UC**) – in Zakopane, Poland, **Ref. 28 (1997), and next, in Refs 34, 53.**

4. MULTICRITERIAL CRITICAL STATES

On multicriterial character of critical states of structures, presented in various tasks by worldwide literature, has turned attention Obrębski in Ref. 45, (2004) Cape Town, South Africa and in Ref. 55, (2008), Itaka, USA. From that times, were started more systematic investigations of influence particular parameters and structure properties on its critical state, Refs 51, 54, (with Tolksdorf) and in Refs 65, 67, 68.

So, it was tested influences of following type variables:

- **v**alue of external loading (Euler, 1707-83),
- length of the bar and slenderness (Euler, 1707-83),
- influence of boundary conditions of structures, Refs. 9, 17, 27,

- influence of material including composite ones, Ref. 27,
- type of bar cross-section, (one of the historically older), Refs 9, 23, 27 (here can be helpful various approaches see Refs 3, 4, 22, 25, 26, 30-33, 36, 39, 61, 62, 65),
- external combined loadings, Refs 54,
- kind of travelling mass, Refs 6, 16, 73, 77,
- mass of the structure, Ref. 23,
- velocity of travelling mass (masses), Refs 71-73.

5. CRITICAL STATES OF THIN-WALLED STRAIGHT BARS

Es it was mentioned, the significant progress in investigation of bars behaviour was done by analysis of *thin-walled bars* (**TWBs**). Especially in range of theory of second order including instability of the bars, were in integral way applied four differential equations as for its bending-torsion behaviour. Here should be pointed some authors (see Ref 23), but the most important is quoted Vlasov's book, Ref. 82. The theory was published or modified in different manner world-wide (for certain reviews see Ref. 23). In the most clear and easy way it was presented by J. Mutermilch (with co-authors), Refs 9, 17.

Important extension of above theory is given in Refs 23, 27, 60, 63 etc. It embraces for thin-walled prismatic bars: statics (in it stability), dynamics - including dynamical stability (both with general dumping – as in Ref. 74), combined loadings (also for longitudinal forces positive and negative), homogenous and composite bar **CSs** etc. Given examples presents: analytical, numerical and hybrid solutions.

6. CRITICAL STATES FOR PLATE STRUCTURES

The list of authors presenting solutions for plates is very long. Here are mentioned such names, only: for plates Z. Kączkowski (analytically and numerically including **FDM**, Ref 15), M. Kwieciński and others, all in range of bending behaviour.

Certain solutions by **FDM** is also presented by present author for mass (force) travelling along landing air belt and on curved path or circular line. So, in such type tasks calculated by **FDM**, is produced **SM** like in Eqn 2. Therefore, we can expect that even here can be found certain critical parameter, e.g. thickness of plate (for certain travelling mass) or velocity of the mass. Such example up to the moment was not tested.

7. CRITICAL STATES OF BRIDGES UNDER MOVING MASSES

The first recognition of the above question was given in authors paper Ref. 44. In two last years' (2017, 2018) present author has turning special attention on behaviour of bridges – its answer on moving loads, Refs 71-73. There, are observed as well displacements of girder (deflections and rotations) as instability phenomenon (values Δ of **SM** - main determinant **K**). For recognition influence of some parameters of the task, were calculated and compared results for at all 312 similar bridges with thin-walled steel cross-section, with a few lengths, with two values of masses, travelling with different uniform velocities (see Tables 1-7).

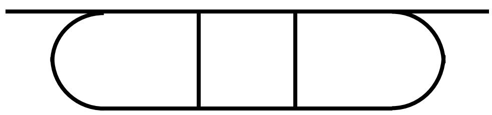

Fig. 1 Cross-section of analyzed bridge, close to reality, Refs 6, 16, 77, and Refs 71-73

As the first step, the task was given for testing by six students: (A. Franus, T. Kleber, J. Kutyna, J. Rawiak, Ł. Rogula, T. Tarabasz e.g. Refs 6, 16, 77) in scope of their homework. In all similar tasks, were investigated the steel girders simply supported, having cross-section as in the Fig. 1, and lengths L=50, 60, 70, 80, 90 and 100m (one in each homework). There, thickness of girder should assure static deflection 0,001 L of bridge span. Moreover, was travelling one concentrated mass Q=20t with uniform velocities in each homework: v=50, 100, 200, 300 and 600 km/h. Despite of certain inadvertences, obtained results confirm expectations and permit on formulation of final assumptions for more serious and large tests – see below.

By application of the 3D-TSM combined with FDM, it is possible to analyse tasks with taking into consideration:

- single mass or set of masses travelling on bridge,
- own mass of structure (bridge) – according to its rigidity,
- velocity of travelling mass,
- length of bridge and its cross-section,
- dumping by interaction of surrounding medium- air, water, liquid.

So, the method seems to be universal, and open wide technical possibilities.

Analysed bridges. The tested bridges have one span simply supported thin-walled girder with **CS** having three closed circuits, Fig. 1. Six tested lengths of bridge girder were: L=50, 60, 70, 80, 90 and 100 m. Over bridge was travelling one concentrated mass with alternatively two values: 20 and 100 t. Thickness of girder **CS** is different and assures approximately static deflections of bridge equal 0,001 L. The girder cross-section is built on square mesh 6x6 m, what gives overall its dimensions 30x6 m, Fig. 1. It is very close to **CSs** used in real bridges.

Assumed velocities of travelling mass were in most cases identical as in students tasks: v=50, 100, 200, 300, and 600 but extended on 1200 km/h, too. Bar was divided on 8 or 10 sections. But in calculated examples with bar division on ten sections and with travelling mass Q=100 t (1000 kN), were calculated more numerous velocities: v= 50, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 1100, 1200, 1500, 1800, 2100, 2400 and 2700 km/h (last velocity higher than 2 speed of sound, see Tables1-7).

Applied theory. In pointed above tasks used for testing bridges, were applied set of four differential equations of four order, derived by present author, Ref. 23. In these equations can be taken into consideration dumping (see Ref. 74) following of girder interaction with surrounding medium (wind pressure, suction, friction and aero-or hydrodynamic effects). In discussed examples, the dumping was omitted. For the specific properties of bridge and taken assumptions, this set of four equations was reduced to two separate equations, finally transformed (Ref. 27) into **FDO**, see Refs 71, 72.

Finite Differences Equations for analysed bridges. The general four differential motion equations from Ref. 23 were step by step simplified (see Ref. 71) and in consequence reduced to two FDO of four order, Egns 4, 5. They are describing dynamics of the bridge, with except of dumping effects and are quoted in Ref. 71. All results obtained by means of these equations are discussed and quoted below.

In Eqns 4, 5, lower indices separated by coma, points actual point in 3D-Time space (t, i). The same point „i" of bar division on sections, in previous time moment is denoted as (t-1, i) and in next time step as (t+1, i). Here $w = v_3$ means deflections and Θ rotation. For other explanations see Ref. 73.

$$\begin{aligned} & w_{t-1,i-1}K_1 + w_{t-1,i}K_2 + w_{t-1,i+1}K_1 + \\ & \quad + w_{t,i-2} + w_{t,i-1}K_3 + w_{t,i}K_4 + w_{t,i+1}K_3 + w_{t,i+2} + \\ & \qquad + w_{t+1,i-1}K_1 + w_{t+1,i}K_2 + w_{t+1,i+1}K_1 = Q \quad , \end{aligned} \tag{4}$$

$$\begin{aligned} & \Theta_{t-1,i-1}G_1 + \Theta_{t-1,i}G_2 + \Theta_{t-1,i+1}G_1 + \\ & \quad + \Theta_{t,i-2} + \Theta_{t,i-1}G_3 + \Theta_{t,i}G_4 + \Theta_{t,i+1}G_3 + \Theta_{t,i+2} + \\ & \qquad + \Theta_{t+1,i-1}G_1 + \Theta_{t+1,i}G_2 + \Theta_{t+1,i+1}G_1 = S \end{aligned} \tag{5}$$

Symbols K_i and G_i means certain coefficients (see Ref.73). In above equations three rows are describing three sequential time moments.

Dynamic stiffness matrix as FDO for analysed bridges. After writing FDO - Eqn 4 and/or Eqn 5 for all points „i" of the bar division, is obtained *dynamic stiffness matrix* (**DSM**), containing information about scheme of girder, its boundary conditions, velocities and positions of vehicles on bridge for all discrete time moments, Fig.2. In consequence the task is coming to solution of the set of linear algebraic eqns type Eqn 2. It has band, symmetrical character. So, in own program DGPST was used upper half band of K (Eqn 2), completed by column of "loadings" Q_i .

In the case, when on each node are used one Eqn 4 or 5, then we apply Eqn 6_1 . But when for each point of the girder are used two or more Eqns, in such case are applied equations type 6_2. So, in the second case we say, that it is used **D**ifference-**M**atrix **E**quation **M**ethod (**DMEM** – description applied in computer program WDKM Ref. 20), see also Refs 21, 23, 37.

Much more detailed explanations of composition Eqn 2 are given in Refs 35, 71, 72, and 79.

$$C_r \sum_{t-1}^{t+1} \left(A_{ro} + \sum_{\Lambda=1}^{n} A_{r\Lambda} E_\Lambda \right) \Phi_r = Q_r \quad \sum_{t-1}^{t+1} \left[\sum_{\Lambda=1}^{N} (W_\Lambda^o + W_\Lambda E_\Lambda) \right] x = q \cdot \tag{6}$$

Space 3D-time from numerical point of view belongs to 2D tasks. There, we have equilibrium state of structure, as Eqn 2, taken together for all time moments (in Fig. 2 for 7 moments).

Time m.	t=1	t=2	t=3	t=4	t=5	t=6	t=7	
t=1	K_r	V_r						Q_1
t=2	V_r	K_r	V_r					Q_2
t=3		V_r	K_r	V_r				Q_3
t=4			V_r	K_r	V_r			Q_4
t=5				V_r	K_r	V_r		Q_5
t=6					V_r	K_r	V_r	Q_6
t=7						V_r	K_r	Q_7

Fig. 2 **DSM** Eqn. 3_1 - K and terms Q for the bar, when m=7, n=7, l=1, by uniform motion. All sub-matrices have dimensions: K_r[7x7], V_r[7x7]

Here matrix **K** as **DSM**, Ref 73 (Mech.) has number of rows and columns equal $N = l \times n \times m$, and l means number equations used for each node (e.g. one to four as Eqns 4, 5); n - number points of girder division; m – number of assumed time moments with time distance Δt. So, each element of global **DSM** - **K**[N×N] is traditional stiffness matrix **K**[w×w], where $w = l \times n$, Refs 71-73. Moreover, boundary conditions for girder are taken into account traditionally (Ref. 27) and for initial time *t=0* (known or equal zero) and final –usually as „back" difference. In Ref 73 are show two variants of **DSM**, when are used Eqns 4, 5 – for bending and torsion (Ref. 73, Figs 5, 6). In the first case is applied system of unknowns w and Θ grouped together – what gives width of half-band m=16 (Fig. 5, for n=8) and mixed system – when for each node displacements w and Θ are used alternately, resulting with m=17 – a little less optimal, (Ref. 73, Fig 6, for n=8).

Values of main determinant for analysed bridges. In Ref 73, are also given a few results presenting calculated values of main determinant for matrix K for analysed bridges. Below, in Tables 1and 2, are given values Δ for all tasks analysed in assumed investigations. In the first case (columns 3, 6, 9) is calculated value of Δ for bridge only. In two next columns are shown values of Δ for 14 (for n=8) or 18 (for n=10) time moments, when loading is throw 7 or 9 time moments moving also after the bridge. In each third case (columns 5, 8, 11) the value of Δ was calculated by own program DGPST (here, for solution of Eqn 2 the symmetrical global stiffness matrix K is remembered as upper half band, only) written by J.B Obrębski. So, the value of Δ is obtained by the way while solving Eqn 2, as set of linear algebraic equations.

Table 1. Values Δ main determinant of stiffness matrix K composed for deflections of bridge loaded by travelling one force (mass) P=200 or 1000 kN and with division of girder on n=8 or 10 sections

Span	Velo-city	Value of Δ by deflection w ; P=200 kN			Value of Δ by deflection w ; P=200 kN			Value of Δ by deflection w ; P=1000 kN		
		n=8	n=8	n=8	n=10	n=10	n=10	n=8	n=8	n=8
L=	v=	Bridge, only	Bridge+after	Bridge+after	Bridge, only	Bridge+after	Bridge+after	Bridge, only	Bridge+after	Bridge+after
m	km/h	MS-Excel	MS-Excel	JO DGPST	MS-Excel	MS-Excel	JO DGPST	MS-Excel	MS-Excel	JO DGPST
1	2	3	4	5	6	7	8	9	10	11
50	50	3,26E+12	1,59E+27	1,06E+25	5,48E+17	3,00E+35	3,00E+35	4,11E+12	2,45E+27	1,69E+25
	100	1,20E+12	2,42E+26	1,42E+24	6,11E+16	3,65E+33	3,65E+33	3,35E+12	1,66E+27	1,12E+25
	200	3,77E+08	**-8,09E+18**	5,67E+16	2,81E+14	**-2,65E+28**	**-2,65E+28**	1,36E+12	3,05E+26	1,83E+24
	300	8,58E+10	1,77E+24	6,60E+21	1,46E+17	1,89E+34	1,89E+34	1,99E+11	8,30E+24	3,75E+22
	600	2,79E+13	**-1,14E+28**	6,45E+25	1,23E+18	8,78E+33	8,80E+33	2,98E+10	3,89E+22	7,19E+20
	1200	**-4,56E+15**	**-6,02E+33**	3,08E+32	**-5,51E+21**	5,30E+44	5,30E+44	1,80E+12	**-1,34E+26**	**-9,66E+23**
60	50	2,86E+12	1,24E+27		4,18E+17	1,74E+35		2,86E+12	1,24E+27	
	100	6,17E+11	6,99E+25		1,13E+16	1,18E+32		6,17E+11	6,99E+25	
	200	**-6,09E+09**	**-1,42E+21**		**-4,39E+15**	**-2,51E+30**		**-6,09E+09**	**-1,42E+21**	
	300	**-6,63E+11**	**-7,14E+25**		**-1,93E+18**	**-1,13E+36**		**-6,63E+11**	**-7,14E+25**	
	600	**-2,83E+12**	1,30E+26		1,52E+20	1,15E+40		**-2,83E+12**	1,30E+26	
	1200	**-1,93E+16**	7,95E+32		3,63E+24	6,57E+50		**-1,93E+16**	7,95E+32	
70	50	2,44E+12	9,20E+26		2,99E+17	8,91E+34		2,44E+12	9,20E+26	
	100	2,46E+11	1,25E+25		4,86E+14	1,72E+29		2,46E+11	1,25E+25	
	200	3,79E+10	7,63E+21		7,85E+16	4,35E+33		3,79E+10	7,63E+21	
	300	**-3,71E+12**	**-2,93E+26**		**-8,93E+18**	7,26E+37		**-3,71E+12**	**-2,93E+26**	
	600	**-5,44E+12**	**-1,72E+29**		**-4,42E+21**	6,94E+42		**-5,44E+12**	**-1,72E+29**	
	1200	5,38E+17	7,50E+35		**-6,01E+26**	4,26E+54		5,38E+17	7,50E+35	
80	50	2,02E+12	6,45E+26		1,99E+17	3,94E+34		2,02E+12	6,45E+26	
	100	6,31E+10	9,92E+23		1,34E+13	8,68E+25		6,31E+10	9,92E+23	
	200	7,77E+10	6,11E+24		9,45E+16	**-1,37E+35**		7,44E+09	**-3,03E+23**	
	300	**-9,34E+10**	**-2,00E+27**		5,99E+17	**-1,93E+38**		**-9,34E+10**	**-2,00E+27**	
	600	3,83E+13	**-2,28E+31**		1,73E+21	1,10E+46		3,83E+13	**-2,28E+31**	
	1200	2,87E+19	4,08E+39		4,90E+28	**-2,14E+56**		2,87E+19	4,08E+39	
90	50	1,61E+12	4,24E+26		1,22E+17	1,46E+34		1,61E+12	4,24E+26	
	100	4,75E+09	8,10E+21		**-3,10E+13**	**-4,27E+27**		4,75E+09	8,10E+21	
	200	**-6,48E+11**	**-7,16E+25**		**-2,08E+18**	**-1,79E+36**		**-6,48E+11**	**-7,16E+25**	
	300	2,40E+13	**-4,30E+28**		8,76E+18	**-5,96E+37**		2,40E+13	**-4,30E+28**	
	600	**-6,28E+15**	1,41E+33		**-9,70E+23**	**-8,76E+47**		**-6,28E+15**	1,41E+33	
	1200	**-6,83E+20**	**-7,19E+40**		**-1,56E+30**	2,65E+58		**-6,83E+20**	**-7,19E+40**	

Table 1. Values Δ main determinant of stiffness matrix K composed for deflections of bridge loaded by travelling one force (mass) P=200 or 1000 kN and with division of girder on n=8 or 10 sections (cont.)

Span	Velo-city	Value of Δ by deflection w ; P=200 kN			Value of Δ by deflection w ; P=200 kN			Value of Δ by deflection w ; P=1000 kN		
		n=8	n=8	n=8	n=10	n=10	n=10	n=8	n=8	n=8
L=	v=	Bridge, only	Bridge+after	Bridge+after	Bridge, only	Bridge+after	Bridge+after	Bridge, only	Bridge+after	Bridge+after
m	km/h	MS-Excel	MS-Excel	JO DGPST	MS-Excel	MS-Excel	JO DGPST	MS-Excel	MS-Excel	JO DGPST
1	2	3	4	5	6	7	8	9	10	11
100	50	1,24E+12	2,60E+26	1,53E+24	6,68E+16	4,36E+33	4,36E+33	3,46E+12	1,77E+27	1,20E+25
	100	9,44E+07	**-1,27E+19**	**-2,32E+17**	3,16E+14	3,68E+28	3,68E+28	1,58E+12	4,05E+26	2,47E+24
	200	**-2,78E+12**	**-4,33E+26**	6,34E+24	**-8,08E+18**	4,53E+37	4,53E+37	3,54E+09	4,62E+21	6,47E+18
	300	5,03E+13	**-4,64E+28**	9,20E+25	**-2,04E+17**	1,70E+34	1,70E+34	1,57E+10	**-3,59E+22**	**-2,23E+20**
	600	**-3,56E+16**	**-4,37E+34**	2,79E+32	**-6,42E+23**	**-1,66E+46**	**-1,66E+46**	1,54E+13	**-1,66E+28**	**-2,44E+26**
	1200	7,62E+20	4,03E+40	1,46E+40	3,68E+32	**-2,68E+62**	4,68E+65	**-3,33E+15**	3,27E+32	**-1,91E+31**

In both tables 1and 2 values of the determinants Δ calculated for mass travelling only on bridge (time moments 1-7 (n=7) or 1-9 (n=10), are printed by normal letters (columns 3, 6, 9). Contrary, the values of Δ for whole task 14 (n=8) or 18 (n=10) time moments, when mass is travelling throw the bridge and identical time period after it, are printed by *italic* letters (columns 4, 5, 7, 8, 10, 11).

Similar mode of distinction is applied in Tables 3 and 4. Moreover, in all Tables 1-4 by **bold** letters are distinguished negative values of determinant Δ. Such outlook of presented results, facilitate to analyse critical states of particular bridges. It should be explained, that each passing through zero value of Δ means critical velocity of mass. Such precise critical velocities were here not calculated.

Table 2. Values Δ of main determinant of stiffness matrix K composed only for torsion of bridge loaded by travelling one force (mass) P=200 or 1000 kN and division of girder on n=8 or 10 sections

Span	Velo-city	Value of Δ by torsion Θ ; P=200 kN			Value of Δ by torsion Θ ; P=200 kN			Value of Δ by torsion Θ ; P=1000 kN		
		n=8	n=8	n=8	n=10	n=10	n=10	n=8	n=8	n=8
L=	v=	Bridge, only	Bridge+after	Bridge+after	Bridge, only	Bridge+after	Bridge+after	Bridge, only	Bridge+after	Bridge+after
m	km/h	MS-Excel	MS-Excel	JO DGPST	MS-Excel	MS-Excel	JO DGPST	MS-Excel	MS-Excel	JO DGPST
1	2	3	4	5	6	7	8	9	10	11
50	50	1,99E+35	5,94E+71		9,69E+47	9,21E+95		4,03E+31	4,43E+64	
	100	**-3,31E+31**	1,00E+64		3,39E+44	1,13E+89		**-1,80E+31**	**-1,46E+62**	
	200	**-9,64E+31**	5,43E+63		3,45E+44	**-2,02E+89**		6,77E+33	**-4,68E+68**	
	300	8,37E+32	2,10E+66		3,14E+45	**-1,78E+90**		2,69E+37	3,88E+70	
	600	2,88E+34	**-3,45E+70**		7,74E+51	**-2,09E+105**		1,39E+62	2,52E+123	
	1200	3,39E+55	1,48E+114		3,03E+78	7,92E+155		5,13E+93	3,00E+184	
60	50	1,36E+40	2,26E+81		6,93E+54	4,71E+109		1,36E+40	2,26E+81	
	100	**-1,76E+36**	4,24E+73		2,22E+51	4,62E+102		**-1,76E+36**	4,24E+73	
	200	**-2,63E+35**	**-1,62E+72**		**-1,43E+51**	**-2,50E+102**		**-2,63E+35**	**-1,62E+72**	
	300	1,63E+37	**-2,36E+74**		**-5,94E+52**	**-1,86E+104**		1,63E+37	**-2,36E+74**	
	600	**-1,74E+39**	1,38E+80		1,83E+57	**-3,66E+115**		**-1,74E+39**	1,38E+80	
	1200	9,76E+64	**-6,35E+129**		**-6,26E+91**	**-1,13E+182**		9,76E+64	**-6,35E+129**	
70	50	4,67E+44	2,27E+90		2,51E+61	6,16E+122		4,67E+44	2,27E+90	
	100	**-4,45E+40**	2,59E+82		7,06E+57	4,29E+115		**-4,45E+40**	2,59E+82	
	200	1,69E+40	**-9,76E+80**		**-5,83E+57**	1,50E+115		1,69E+40	**-9,76E+80**	
	300	5,05E+40	**-3,02E+80**		**-1,68E+58**	7,53E+116		5,05E+40	**-3,02E+80**	
	600	**-2,84E+43**	**-1,41E+86**		**-4,98E+63**	8,88E+126		**-2,84E+43**	**-1,41E+86**	
	1200	**-1,25E+72**	1,59E+141		**-2,34E+102**	**-9,72E+205**		**-1,25E+72**	1,59E+141	
80	50	7,08E+48	4,66E+98		3,57E+67	1,25E+135		7,08E+48	4,66E+98	
	100	**-5,08E+44**	2,88E+90		8,85E+63	6,23E+127		**-5,08E+44**	2,88E+90	
	200	2,12E+44	7,70E+88		**-5,59E+63**	1,75E+127		2,12E+44	7,70E+88	
	300	**-3,81E+44**	**-7,41E+87**		2,39E+64	2,98E+127		**-3,81E+44**	**-7,41E+87**	
	600	**-9,97E+47**	1,41E+95		**-1,49E+69**	**-3,82E+136**		**-9,97E+47**	1,41E+95	
	1200	**-6,40E+77**	**-1,33E+153**		4,47E+112	2,11E+223		**-6,40E+77**	**-1,33E+153**	
90	50	5,18E+52	2,29E+106		2,12E+73	4,38E+146		5,18E+52	2,29E+106	
	100	**-2,87E+48**	7,02E+97		4,70E+69	1,63E+139		**-2,87E+48**	7,02E+97	
	200	9,92E+47	4,57E+96		**-2,07E+69**	2,38E+138		9,92E+47	4,57E+96	
	300	**-2,16E+48**	**-7,16E+95**		7,01E+69	**-2,42E+138**		**-2,16E+48**	**-7,16E+95**	
	600	1,41E+53	1,09E+104		**-6,03E+74**	1,43E+147		1,41E+53	1,09E+104	
	1200	**-1,08E+82**	**-4,47E+163**		6,46E+119	**-4,43E+239**		**-1,08E+82**	**-4,47E+163**	
100	50	2,24E+56	4,00E+113		6,70E+78	4,39E+157		3,14E+51	**-2,03E+106**	
	100	**-9,84E+51**	4,74E+104		1,35E+75	1,27E+150		**-1,57E+51**	**-1,53E+104**	
	200	2,46E+51	5,33E+103		**-4,14E+74**	8,90E+148		2,79E+51	**-1,04E+105**	
	300	**-3,86E+51**	1,73E+103		7,20E+74	**-2,26E+148**		**-5,96E+59**	1,07E+122	
	600	5,42E+55	2,31E+113		**-1,19E+80**	**-5,74E+158**		1,93E+90	**-1,96E+182**	
	1200	8,46E+86	3,51E+172		**-7,79E+127**	2,26E+253		1,71E+122	**-1,40E+242**	

Comments to critical states of bridges under moving masses. More detailed observations of Tables 1-4 permit to conclude, that it is serious difference in values of Δ obtained by commercial program MS Excel and own DGPST written in RM Fortran. Such differences can occur as result of other length of remembered “words” by computer.

Moreover, more recommended are rather critical values calculated for mass acting on bridge, only (columns 3, 6, 9). Values of Δ quoted in columns 5, 8, 11 were obtained by calculation deflection lines for bridge, when mass is moving uniformly through bridge and some way after. It permit to observe “answer” of girder (deflections) also after it passing, as in the Fig. 8. For limited volume even these paper, such diagrams are here not presented.

As it was shown in the work Ref. 20, on the ground of numerical test of own program WDKM in range theory of I-st and the II-nd order, compared with analytical exact Euler’s solution, that after passing critical value of longitudinal compressing force, displacements grow up to infinity (see Eqn 1) and the task show losing of equilibrium state reactions and external loading. So, results given in Tables 1-4should be analysed together with calculated deflection lines of travelling loadings, too.

In calculated examples, were compared results for bridges simply supported with spans L=50, 60,70, 80, 90 and 100 m. Shown results permit to conclude, that on velocity of travelling mass are more impressionable longer bridges.

To the program of comparative examples were included calculations of similar tasks, when the bridge girder is divided on n=8 or n=10 sections and its influence on values obtained critical velocities. It is evident, that more dense division should assure better result. But here, in case of calculation of values Δ, appears problem with magnitude of obtained numbers. So, density of division should be carefully selected and limited. As it follows, especially from Table 4, the values of torsion of girder are much bigger then for bending. There, for velocity of mass bigger of 800 kN/h program MS Excel display announcement about to big numbers (indicate “#NUMBER!”). But when applying program DGPST such problem is not observed.

Table 3. Values of main determinant of stiffness matrix K **for deflections** of bridge loaded by travelling one force (mass) P= 1000 kN and division of girder on n=10 sections

Span	Velo-city	Value of Δ by deflection w ; P=1000		
		n=10	n=10	n=10
L=	v=	Bridge, only	Bridge+after	Bridge +after
m	km/h	MS-Excel	MS-Excel	JO DGPST
1	2	3	4	5
50	50	8,74E+17	7,64E+35	7,64E+35
	100	5,76E+17	3,32E+35	3,32E+35
	200	8,28E+16	6,74E+33	6,73E+33
	300	2,33E+14	3,66E+28	3,17E+62
	600	3,47E+16	9,94E+32	9,94E+32
	1200	**-5,69E+16**	**-7,50E+32**	**-7,50E+32**
60	50	4,18E+17	1,74E+35	
	100	1,13E+16	1,18E+32	
	200	**-4,39E+15**	**-2,51E+30**	
	300	**-1,93E+18**	**-1,13E+36**	
	600	1,52E+20	1,15E+40	
	1200	3,63E+24	6,57E+50	
70	50	2,99E+17	8,91E+34	
	100	4,86E+14	1,72E+29	
	200	7,85E+16	4,35E+33	
	300	**-8,93E+18**	7,26E+37	
	600	**-4,42E+21**	6,94E+42	
	1200	**-6,01E+26**	4,26E+54	
80	50	2,24E+11	4,47E+21	
	100	**-1,22E+09**	4,81E+16	
	200	1,97E+11	**-3,21E+21**	
	300	1,84E+12	**-3,02E+23**	
	600	2,71E+14	1,03E+28	
	1200	**-1,29E+18**	1,03E+35	

Table 3. Values of main determinant of stiffness matrix K **for deflections** of bridge loaded by travelling one force (mass) P= 1000 kN and division of girder on n=10 sections (cont.)

Span	Velo-city	Value of Δ by deflection w ; P=1000		
		n=10	n=10	n=10
L=	v=	Bridge, only	Bridge+after	Bridge +after
m	km/h	MS-Excel	MS-Excel	JO DGPST
1	2	3	4	5
90	50	1,70E+11	2,54E+21	
	100	2,30E+09	3,93E+17	
	200	8,69E+11	2,40E+22	
	300	3,93E+11	**-1,23E+23**	
	600	**-4,42E+14**	**-4,89E+28**	
	1200	1,91E+18	**-5,66E+34**	
100	50	6,17E+17	3,81E+35	3,81E+35
	100	1,15E+17	1,32E+34	1,32E+34
	200	**-1,93E+13**	**-4,94E+27**	**-4,94E+27**
	300	3,54E+16	1,66E+32	1,66E+32
	400	**-1,55E+18**	**-6,74E+35**	**-6,74E+35**
	500	**-3,67E+18**	4,67E+36	4,67E+36
	600	1,03E+18	**-5,13E+35**	**-5,13E+35**
	700	**-3,22E+18**	2,01E+36	2,01E+36
	800	1,39E+20	1,10E+40	1,10E+40
	900	**-1,18E+21**	**-3,15E+39**	**-3,18E+39**
	1000	**-2,20E+21**	2,79E+42	2,79E+42
	1100	**-1,34E+22**	5,67E+44	5,67E+44
	1200	**-3,64E+22**	**-1,18E+45**	**-1,18E+45**
	1500	**-2,41E+24**	**-8,37E+47**	**-8,37E+47**
	1800	**-2,30E+26**	3,72E+51	3,72E+51
	2100	2,19E+25	3,77E+52	3,77E+52
	2400	**-4,60E+28**	1,32E+57	1,32E+57
	2700	1,72E+28	8,90E+54	8,89E+54

Displacements of analysed bridges under moving masses. In calculated examples was foreseen observation of displacements lines of analysed bridges. Such comparisons were limited to deflections w (Eqn 4) and rotations – torsion angles Θ (Eqn 5). From numerical point of view, they both are independent. But from physical point of view, they should be considered and analysed commonly. Here also from limited volume of this paper, presented results are in Tables 5 to 7 strongly limited. There, are shown only position of moving mass, and points in the tasks, where displacement obtains maximal and minimal values. Detailed comments to showing results are here not given.

It is worthy to turn the attention, that only for girder with length L=100 m divided on n=10 sections, results of calculated displacements are show for mass velocity up to v=2700 km/h – it exceed much over 2 times speed of sound, and is rather not noted in civil - bridge engineering.

Table 4. Values of main determinant of stiffness matrix K **for torsion** of bridge loaded by travelling one force (mass) P= 1000 kN and division of girder on n=10 sections

Span	Velo-city	Value of Δ by torsion Θ ; P=1000		
		n=10	n=10	n=10
L=	v=	Bridge, only	Bridge +after	Bridge +after
m	km/h	MS-Excel	MS-Excel	JO DGPST
1	2	3	4	5
50	50	**-5,55E+44**	**-3,45E+89**	**-3,45E+89**
	100	**-2,63E+45**	5,43E+89	5,43E+89
	200	**-3,99E+48**	**-2,92E+97**	**-2,92E+97**
	300	**-1,29E+53**	**-6,55E+102**	**-6,55E+03**
	600	**-9,36E+86**	**-2,13E+172**	**-2,13E+73**
	1200	8,14E+136	1,43E+272	1,44E+74

Table 4. Values of main determinant of stiffness matrix K **for torsion** of bridge loaded by travelling one force (mass) P= 1000 kN and division of girder on n=10 sections (cont.)

Span	Velo-city	Value of Δ by torsion Θ ; P=1000		
		n=10	n=10	n=10
L=	v=	Bridge, only	Bridge +after	Bridge +after
m	km/h	MS-Excel	MS-Excel	JO DGPST
1	2	3	4	5
60	50	6,93E+54	4,71E+109	
	100	2,22E+51	4,62E+102	
	200	**-1,43E+51**	**-2,50E+102**	
	300	**-5,94E+52**	**-1,86E+104**	
	600	1,83E+57	**-3,66E+115**	
	1200	**-6,26E+91**	**-1,13E+182**	
70	50	2,51E+61	6,16E+122	
	100	7,06E+57	4,29E+115	
	200	**-5,83E+57**	1,50E+115	
	300	**-1,68E+58**	7,53E+116	
	600	**-4,98E+63**	8,88E+126	
	1200	**-2,34E+102**	**-9,72E+205**	
80	50	2,24E+11	4,47E+21	
	100	**-1,22E+09**	4,81E+16	
	200	1,97E+11	**-3,21E+21**	
	300	1,84E+12	**-3,02E+23**	
	600	2,71E+14	1,03E+28	
	1200	**-1,29E+18**	1,03E+35	
90	50	2,12E+73	4,38E+146	
	100	4,70E+69	1,63E+139	
	200	**-2,07E+69**	2,38E+138	
	300	7,01E+69	**-2,42E+138**	
	600	**-6,03E+74**	1,43E+147	
	1200	6,46E+119	**-4,43E+239**	
100	50	**-2,35E+75**	3,03E+150	3,03E+51
	100	7,49E+73	3,44E+146	3,44E+47
	200	**-5,75E+75**	**-5,70E+151**	**-5,70E+52**
	300	7,91E+84	**-5,85E+171**	**-5,85E+72**
	400	4,47E+104	**-9,66E+209**	1,72E+63
	500	9,57E+122	1,38E+244	1,38E+46
	600	8,87E+134	6,67E+270	6,67E+72
	700	2,54E+147	3,97E+292	3,97E+94
	800	**-6,75E+156**	#NUMBER!	**-6,32E+13**
	900	6,97E+164	#NUMBER!	**-2,41E+32**
	1000	1,54E+174	#NUMBER!	2,55E+47
	1100	2,78E+180	#NUMBER!	8,49E+59
	1200	**-2,43E+187**	#NUMBER!	**-3,66E+73**
	1500	5,20E+201	#NUMBER!	3,34E+06
	1800	**-4,33E+216**	#NUMBER!	**-2,86E+33**
	2100	**-1,76E+229**	#NUMBER!	**-4,22E+54**
	2400	**-2,38E+239**	#NUMBER!	4,40E+74
	2700	**-1,25E+248**	#NUMBER!	**-1,55E+91**

For empty cells in Tables 2-4, were not calculated displacement lines and values Δ .

Table 5. Deflections w of bridges – positions of loading with maximums and minimums; n=8

Force	Span	Velo-city	Posi-tion P	Deflection		Posi-tion P	Deflection	
P=	L=	v=	T=	Point	Max.	T=	Point	Min.
kN	m	km/h	Nr.	Nr.	m	Nr.	Nr.	m
200	50	50	4	4	0,05103	8	4	-0,00037
		100	4	4	0,05146	8	4	-0,00173
		200	5	4	0,123	9	4	-0,102
		300	5	4	0,054	9	4	-0,028

Table 5. Deflections w of bridges – positions of loading with maximums and minimums; n=8 (cont.)

Force	Span	Velo-city	Posi-tion P	Deflection		Posi-tion P	Deflection	
P=	L=	v=	T=	Point	Max.	T=	Point	Min.
kN	m	km/h	Nr.	Nr.	m	Nr.	Nr.	m
		600	7	4	0,337	12	4	-0,333
		1200	**10**	4	0,064	2	6	-0,014
	100	50	4	4	0,099	8	4	-0,003
		100	3	4	0,136	13	4	-0,012
		200	3	4	0,114	10	4	-0,055
		300	7	4	0,208	2	4	-0,209
		600	**9**	7	0,141	1	6	-0,065
		1200	2	7	0,016	7	5	-0,070
1000	50	50	4	4	0,051	8	4	-0,000079
		100	4	4	0,051	8	4	-0,000328
		200	4	4	0,051	8	4	-0,001
		300	4	4	0,052	8	4	-0,005
		600	4	4	0,063	14	4	-0,026
		1200	3	4	0,173	8	4	-0,093
	100	50	4	4	0,098	8	4	-0,000573
		100	4	4	0,099	8	4	-0,00257
		200	4	4	0,416	5	4	-0,373
		300	5	4	0,189	12	4	-0,071
		600	3	4	0,225	8	4	-0,072
		1200	**10**	4	0,147	3	5	-0,059

8. GRAPHICAL INTERPRETATIONS OF CRITICAL STATES

Dependently on type of question given for particular technical solution, appears various needs for graphical interpretation of calculated results. Some examples are given in the Figs 3-8. They presents the more interesting and representative diagrams.

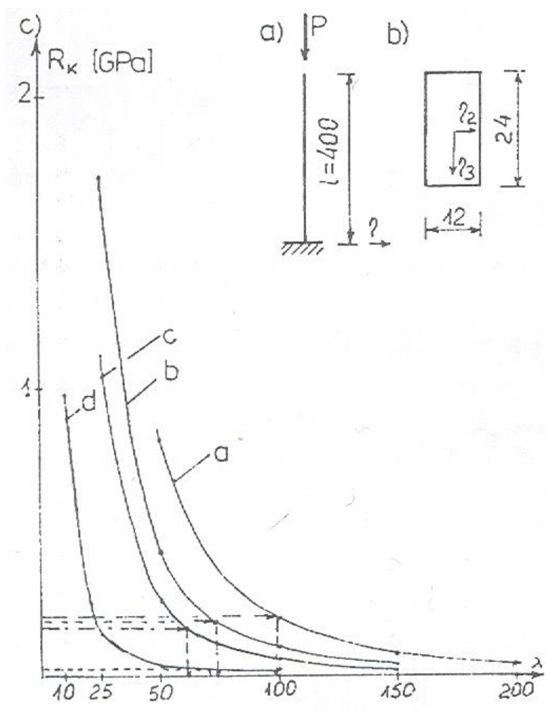

Fig. 3 Diagram of critical stresses in function of slenderness for homogenous bar, made from some different materials: a) steel, b) brazen c) aluminium, d) timber (see Ref. 27)

Table 6. Deflections w of bridges by division of the girders on n=10 sections – points position of loading and maximums and minimums

Force	Span	Velo-city	Posi-tion P	Deflection		Posi-tion P	Deflection	
P=	L=	v=	T=	Point	Max	T=	Point	Min.
kN	m	km/h	Nr.	Nr.	m	Nr.	Nr.	M
200	50	50	5	5	0,050	10	5	-0,00047
		100	5	5	0,051	10	5	-0,00246
		200	3	5	0,121	16	5	-0,068
		300	6	5	0,051	11	5	-0,026
		600	9	5	1,129	16	5	-1,153
		1200	**13**	5	0,064	2	7	-0,016
	100	50	5	5	0,098	10	5	-0,005
		100	5	5	0,173	18	5	-0,101
		200	3	5	0,114	13	5	-0,051
		300	3	5	2,176	10	5	-1,977
		600	**11&14**	3&7	0,184	5	7	-0,092
		1200	**15**	1	0,066	4	1	-0,059
1000	50	50	5	5	0,050	10	5	-0,000099
		100	5	5	0,050	10	5	-0,00041
		200	5	5	0,051	10	5	-0,002
		300	8	5	0,005	2	2	-0,006
		600	5	5	0,056	12	5	-0,028
		1200	3	5	0,151	10	5	-0,075
	100	50	5	5	0,097	10	5	-0,0007
		100	5	5	0,098	10	5	-0,004
		200	4	5	0,127	15	5	-0,008
		300	4	5	0,355	12	5	-0,271
		400	6	5	0,259	17	5	-0,141
		500	7	5	0,318	12	5	-0,277
		600	4	5	0,211	10	6	-0,065
		700	9	5	0,256	16	5	-0,254
		800	7	5	0,161	15	5	-0,146
		900	1	3	2,385	16	3	-2,428
		1000	**14**	5	3,210	5	5	-3,143
		1100	**14**	5	0,250	4	6	-0,182
		1200	**13**	5	0,152	4	6	-0,067
		1500	**10**	3	0,140	2	8	-0,064
		1800	**10**	5	0,262	1	5	0,035
		2100	1	9	-0,012	9	5	-0,342
		2400	**16**	2	0,020	9	4	-0,114
		2700	**16**	9	0,506	10	5	-0,543

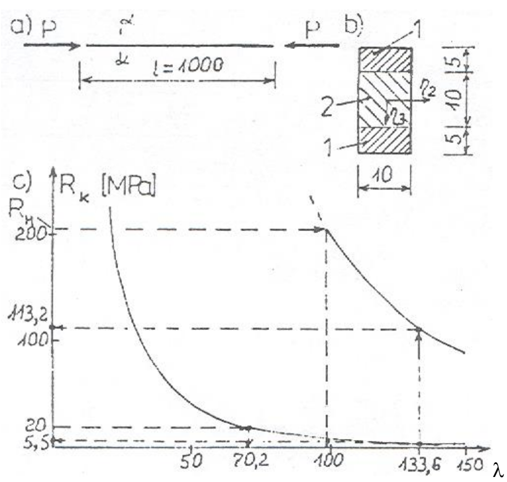

Fig. 4 Diagram of critical stresses in function slenderness λ for analysis of ultimate (global) slenderness for whole compressed bar composed from steel ((1) and timber (2) (see Ref.27)

So, diagrams from Figs 3 and 4 are based on the most popular up to now Euler's solutions for compressed bars. But in the second case its cross-section is composed of two materials.

Table 7. Torsion angles Θ of girder cross-section, for force P=1000 kN. Position force P and places of maximums and minimums, by bar division on n=10.

Span	Velo-city	Posi-tion P	Rotation		Posi-tion P	Rotation		Rotation	
L=	v=	T=	Point	Max.	T=	Point	Min.	Max.	Min.
m	km/h	Nr.	Nr.	rad	Nr.	Nr.	rad	Degree	Degr.
50	50	3	4	0,011	18	5	-0,003	0,63	-0,19
	100	3	3	0,022	12	3	-0,016	1,26	-0,92
	200	5	5	0,049	15	5	-0,044	2,82	-2,53
	300	**10**	8	1,553	3	7	-1,496	88,98	-85,7
	600	**18**	4	-0,00064	4	4	-0,003	-0,036	-0,17
	1200	**14**	2	0,00023	8	6	-0,0008	0,013	-0,05
100	50	5	5	0,006	16	5	-0,002	0,318	-0,12
	100	5	7	0,009	12	7	-0,008	0,535	-0,46
	200	5	5	0,063	14	5	-0,060	3,587	-3,44
	300	**15**	6	0,005	4	4	-0,002	0,265	-0,11
	400	**18**	5	0,00092	10	5	-0,022	0,053	-1,24
	500	**18**	9	-0,00008	8	5	-0,003	-0,004	-0,17
	600	**15**	9	0,00018	9	4	-0,0015	0,011	-0,09
	700	**11**	9	0,000363	9	3	-0,0014	0,021	-0,08
	800	**10**	8	0,002799	9	3	-0,003	0,160	-0,19
	900	**11**	2	0,000505	7	7	-0,001	0,029	-0,06
	1000	**10**	2	0,000242	7	6	-0,001	0,014	-0,03
	1100	**13**	2	0,000187	**8**	6	-0,0005	0,011	-0,03
	1200	**10**	2	0,000517	**9**	5	-0,0007	0,030	-0,04
	1500	**12**	4	0,000056	7	7	-0,0003	0,003	-0,02
	1800	**10**	6	0,000036	8	8	-0,0002	0,002	-0,01
	2100	**10**	5	0,001062	9	7	-0,0012	0,061	-0,07
	2400	**11**	3	0,000023	8	8	-0,0001	0,001	-0,006
	2700	**10**	1	0,00002	8	8	-0,00017	0,001	-0,004

Next, in the Fig 5 is shown diagram of critical *ultimate surface*, for bar combined 3 external loadings: P – longitudinal compressing force and two bending moments.

In the Fig 6 is given example of *ultimate critical izo-surface* for compressed bar. Safe zone is inside (below) calculated diagram.

The Fig 7 presents diagram of critical curves for eccentrically compressed bar. Here, is possible critical state by bar tension, too.

The Fig 8 shows two deflections of bridge in 11 time moments. The first for mass travelling with low velocity and regular deflections, and the second with very high, over critical velocity, with chaotic vibrations (deflections).

Deflections of analysed above bridges discussed in the chapter 7, are similar to that shown in the Fig 8.

The diagrams interpreting data presented in the Tables1-4, will be published by other occasion.

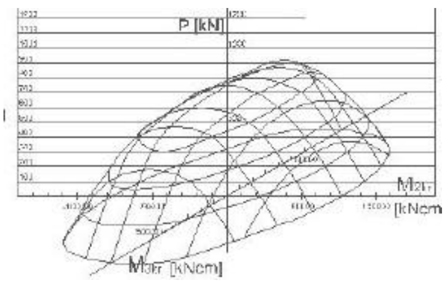

Fig. 5 Critical ultimate surfaces 3D for straight bar with length 400 cm with rectangular composite (steel and timber) cross-section, under action of combined loadings: P, M_2 and M_3. Safe zone is inside this surface (see Refs 68, 69).

The other, next numerous examples of various diagrams can be found in papers with review character, Refs 56, 60, 64, 70.

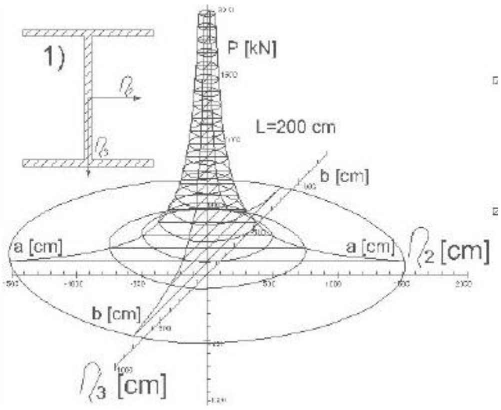

Fig. 6 Ultimate critical izo-surface for the compressed straight bar with I cross-section and with length L=200 cm, (see Ref. 68, 69). The diagram indicates critical force position and its value.

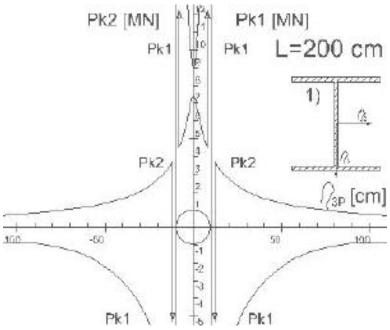

Fig. 7 Diagrams of critical forces P_{k1} , P_{k2} , and $P_{k3}=P_2$. The last is much bigger from critical force $P_{kr}=P_{k2}$. The bar has the length L=200 cm and I cross-section (see Refs 68, 69). The bar is compressed eccentrically by force located on vertical axis of symmetry

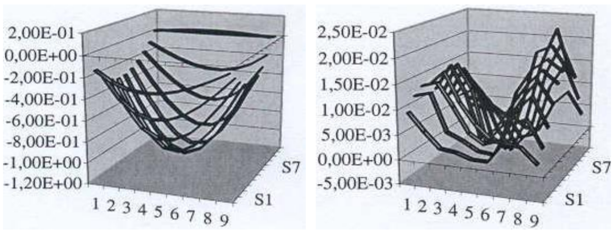

Fig. 8 Comparison of deflections of bridge girder for 11 time moments under mass 100 t travelling with two velocities: 36 and 3600 km/h (see Refs 44, 45, 56, 71). In the 11-th moment loading is after the bridge. The task is very similar to discussed above in chapter 7

9. CONCLUSIONS

Presented in this paper results very wide investigations over behaviour and critical states of bridges, confirm rightness of theses formulated by present author in *Uniform Criterion*. It replenishes very systematic long tests executed through many years documented in author's publications since about 1979. Simultaneously, it proves possibility to apply it to determination of critical states of bridges under moving loads.

In above text were not discussed wider confirmations of numerical results by experiments, Refs 1, 50, 60.

Here it is worthy to indicate three papers oriented on stability problems, giving in easy way a few valuable examples – Refs 46, 47, 69. They extend described above topics. Also, can be recommended papers Refs 50, 51, 53, 58, 71 as important supplement of presented above material.

As the last, yet once should be turned attention on precision of numerical calculations, what is discussed relatively frequently in literature e.g. Refs 47, 76. So, on presented Tables 1-7 we can observe different calculated values. The most visible cases appear in Table 4 for Δ . There, commercial program MS Excel can not to calculate Δ , but contrary own program DGPST has given expected value.

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